

Finite-time scaling of dynamic quantum criticality

Shuai Yin,^{*} Xizhou Qin, Chaohong Lee,[†] and Fan Zhong[‡]

*State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering,
Sun Yat-sen University, Guangzhou 510275, People's Republic of China*

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Time plays a fundamental role in quantum criticality owing to the interplay of static and dynamic behaviors. Yet, it is a great challenge to study and understand real-time quantum critical dynamics both at zero temperature and at finite temperatures. Here, we consider the competition among an external time scale, an intrinsic reaction time scale and an imaginary time scale arising respectively from an external driving field, the fluctuations of the competing orders and thermal fluctuations and explore a regime of finite-time scaling. Through a successful application in the dark impulse regime of quantum Kibble-Zurek mechanism at zero temperature and the first solution of real-time Lindblad master equation near a quantum critical point at nonzero temperatures, we show that finite-time scaling offers not only an amenable and systematic approach to detect and determine dynamic critical properties, but also a unified framework to understand and explore the nonequilibrium dynamics of quantum criticality.

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Detecting and understanding dynamics of quantum phase transitions (QPTs) are of great challenge and importance [1–5]. Recent experimental breakthrough in ultracold atoms [6] promises new tools to study quantum critical dynamics [7]. Different from classical phase transitions whose static and dynamic fluctuations are independent, “time” plays a fundamental role in quantum criticality owing to the interplay of static and dynamic behaviors. A continuous quantum phase transition (QPT) taking place at zero temperature is dominated by a reaction time τ_s , which arises from the fluctuations of the competing orders and blows up as $\tau_s \sim |g|^{-\nu z}$ with the standard critical exponents ν and z as the distance to the critical point g vanishes [1]. It describes how fast the system reacts to external perturbations. A protocol [4, 5] in which the effect of τ_s is clearly exhibited is to drive a system through its critical point by increasing g , say, with a time rate of R . In this case, there exists a characteristic time scale associated with R^{-1} . This is a driving time scale $\tau_d \sim R^{-z/r}$ with a rate critical exponent r that is related to z and other static critical exponents depending on which controlling variable is being swept [20]. Consequently, no matter how small R is, the adiabatic evolution for $\tau_s < \tau_d$ breaks down at a freeze-out time instant $-\hat{t}$ and resumes at \hat{t} with $\hat{t} \sim \tau_d$. In this adiabatic-impulse-adiabatic approximation of the quantum Kibble-Zurek mechanism (KZM) [8–11], the condition for the breakdown of adiabaticity was employed to reckon a famous KZ scaling (see below); while the state at $-\hat{t}$ is assumed to be frozen until \hat{t} , at which it is reactivated and serves as the initial state for the subsequent evolution. Apparently, however, this is not the real situation, as for example, dynamical scaling has been reported within this impulse regime [13, 14].

Natural systems and their measurements exist inevitably in nonzero temperatures, though probably only initial thermal states need considering in ultracold atoms

[15–18]. The finite temperature T introduces an “imaginary” time scale $\tau_T = 1/T$ (in units in which the Planck and the Boltzmann constants are set to 1) since the real time is its analytical continuation to imaginary numbers through a Feynman path integral representation [1]. Whereas for $\tau_T > \tau_s$, thermal effects can be regarded as perturbation to the quantum ground states and conventional quasiparticle approaches are usually effective; for $\tau_T < \tau_s$, thermal fluctuations dominate. Yet, it is this latter quantum critical regime (QCR) that is suggested to the arena of exotic behavior exhibited in a wide range of strongly correlated systems [1–3, 12]. As both phases exhibit complex long-range quantum entanglement owing to the long τ_s and are violently excited thermally in the QCR, new approaches have to be invoked.

Here we systematically study quantum critical behavior due to the interplay among the three time scales τ_s , τ_d and τ_T according to the theory of finite-time scaling (FTS) [19, 20]. As seen in Fig. 1, besides the usual equilibrium regimes and the QCR which are respectively dominated by τ_s and τ_T , the most important consequence is that a new nonequilibrium FTS domain is created. In the impulse regime of the KZM at $T = 0$, although the system falls out of equilibrium, the state does not cease evolving; rather, it evolves according to the imposed time scale τ_d instead of τ_s . As a consequence, nonadiabatic excitations obey FTS just within the dark impulse regime, which overlaps the FTS regime. Therefore, FTS improves the understanding of KZM on its dark impulse regime. Similarly, at $T \neq 0$, in the FTS regime, there are now nonadiabatic thermal excitations controlled again by τ_d and thus obeying again FTS. This then provides an extra excess to detect and study quantum criticality at finite temperatures. Owing to its conceptual simplicity and its accessibility, FTS therefore provides a unified framework not only to determine dynamic critical properties, but also to understand and explore the nonequilibrium dy-

namics of quantum criticality both at $T = 0$ and $T \neq 0$.

In the following, we shall first develop the theory of FTS for QPTs. Then we shall exemplify the determination of the critical point and critical exponents in the impulse regime at $T = 0$ using the paradigmatic quantum Ising model. Finally we shall verify FTS by solving for the first time the Lindblad master equation [21, 22] near the quantum critical point at finite temperatures, which also shows that this equation is an appropriate platform for studying real-time dynamic quantum criticality.

RESULTS

FTS theory for QPTs. Just as the well-known regime of finite-size scaling in which the characteristic size L of a system is shorter than its correlation length ξ , an FTS regime arises when τ_d is the shortest time among the three time scales and thus dominates. The theory of FTS [19, 20] takes explicitly into account the rate R , which plays a role similar to L^{-1} since it imposes on a system an additional time scale that manipulates its evolution. For a length rescaling of factor b , an order parameter M transforms as

$$M = b^{-\beta/\nu} M(tb^{-z}, gb^{1/\nu}, h_z b^{\beta\delta/\nu}, Tb^z, Rb^r, L^{-1}b), \quad (1)$$

where the two critical exponents β and δ are defined as usual in classical critical phenomena by $M \propto g^\beta$ in the absence of an external probe field h_z conjugate to M and $M \propto h_z^\delta$ at $g = 0$, respectively. Equation (1) in its classical form in which g is replaced by $T - T_c$ (T_c is the classical critical point) can be derived from the renormalization-group theory [19, 20]. Here we apply it to the quantal case as critical singularities exhibit too albeit due to quantum rather than thermal fluctuations.

With equation (1), one can describe in a unified framework different kinds of driven dynamics via changing g , h_z or T and readily define different regimes and their crossovers. Taking for example $g = Rt$, neglecting h_z and choosing b such that Rb^r becomes a constant, one finds an FTS scaling form

$$M = R^{\beta/\nu r} f_1(gR^{-1/\nu r}, TR^{-z/r}, L^{-1}R^{-1/r}), \quad (2)$$

where $r = z + 1/\nu$ obtained from $g = Rt$ and its rescaling [19, 20] and the function f_i with an integer i denotes a scaling function. FTS dominates when $|g|R^{-1/\nu r} \ll 1$, $TR^{-z/r} \ll 1$ and $L^{-1}R^{-1/r} \ll 1$. The first is $\tau_d \ll |g|^{-\nu z} \sim \tau_s$ and the second $\tau_d \ll 1/T = \tau_T$ as they should be. Crossovers to other regimes occur near $|\hat{g}| \sim R^{1/\nu r}$ and $T \sim R^{z/r}$ as depicted in Fig. 1 and a similar one for L . The first gives $|\hat{t}| \sim R^{-\nu z/(1+\nu z)}$ because $\hat{g} = R\hat{t}$. This is just the result of the KZM [4, 5, 8–10] in the thermodynamic limit ($L \rightarrow \infty$) and at $T = 0$.

Scaling for other observables can be simply written down. The density of quasiparticle excitations n_{ex} , for

instance, is

$$n_{\text{ex}} = R^{d/r} f_2(gR^{-1/\nu r}, TR^{-z/r}, L^{-1}R^{-1/r}), \quad (3)$$

as it is proportional to ξ^{-d} in a d -dimensional space. (3) gives rise to the leading singularity $n_{\text{ex}} \propto R^{d/r}$, which is the famous scaling of KZM [4, 5, 8–10], or more precisely, $n_{\text{ex}} = R^{d/r} f_2(gR^{-1/\nu r}) = R^{d/r} f_2(t/\hat{t})$ for $T = 0$ in the thermodynamic limit in agreement with that suggested previously [4, 13]. It also predicts that $n_{\text{ex}} \propto T^{d/z}$ and $n_{\text{ex}} \propto L^{-d}$ with appropriate conditions in the QCR and finite-size scaling regime, respectively. Comparing with the scaling forms in the imaginary-time quench [23], here we directly focus on the real-time behavior which is realizable directly in experiments. Different from similar expressions including real-time dynamics [24, 25] and the finite temperature effects [14], one sees that FTS provides a unified framework not only to understand the nonequilibrium evolution, but also to derive various scaling forms. Indeed, we can readily consider other kinds of sweepings near the critical point as follows.

Instead of sweeping g , when $h_z = R_z t$, one replaces R with R_z and obtains similarly the order parameter

$$M_h = R_z^{\beta/\nu r_z} \times f_3(gR_z^{-1/\nu r_z}, h_z R_z^{-\beta\delta/\nu r_z}, TR_z^{-z/r_z}, L^{-1}R_z^{-1/r_z}) \quad (4)$$

with $r_z = z + \beta\delta/\nu$ now. Different regimes and their crossovers can be readily defined in this case too. Different from sweeping the quadratic coupling term, here we change the symmetry breaking field h_z and we shall find this sweeping provides an experimentally realizable method to determine the critical point.

We can also vary T with a rate R_T , i.e., $T = R_T t$, the scaling form for the order parameter is then

$$M_T = R_T^{\beta/\nu r_T} \times f_4(gR_T^{-1/\nu r_T}, h_z R_T^{-\beta\delta/\nu r_T}, TR_T^{-z/r_T}, L^{-1}R_T^{-1/r_T}) \quad (5)$$

with $r_T = 2z$. Theoretically, according to Feynman path integral representation [1], varying T is equivalent to changing the imaginary time scale τ_T . Experimentally, varying temperature can even be realized in cold atoms experiments [26].

According to these scaling analyses, the regime dominated by τ_d , i.e., the FTS regimes can then be determined. For changing g , a schematic phase diagram is shown in Fig. 1. At $T = 0$, the FTS regime is thus just the impulse regime of the KZM. As a result, hysteretic dynamics in this regime is expected to be describable by FTS. One sees from Fig. 1 that the FTS regime pushes the QCR to higher temperatures, because only then τ_T dominates. Consequently, the FTS enables one to probe directly the quantum critical point and its scaling behavior at nonzero temperatures as the temperature becomes subordinate, i.e., just a perturbation. In other words, the nonequilibrium driving appears to

extend the zero-temperature critical point to the whole FTS regime, or conversely, has a ‘dynamic cooling’ effect that enables one to probe zero-temperature scaling at nonzero-temperatures while keeping the temperature subsidiary. This offers us an extra approach to detect and study quantum criticality at finite temperatures. Therefore, FTS provides not only a unified, simple and feasible approach to test predictions of microscopic theories but also a systematic approach to study equilibrium and nonequilibrium dynamics of quantum criticality.

Determination of critical properties. Here we show that FTS can also provide methods to detect quantum critical properties such as the critical point and critical exponents. As stated above, in the FTS region, effects of the temperature can be neglected, so we consider $T = 0$ in the thermodynamic limit $L = \infty$. According to equation (4), the external field at $M_h = 0$, denoted by h_{zm} , is scaled as

$$h_{z0}(g, R_z) = R_z^{\beta\delta/\nu r_z} f_5(g R_z^{-1/\nu r_z}). \quad (6)$$

At $h_z = 0$, M_h is reduced to

$$M_0(g, R_z) = R_z^{\beta/\nu r_z} f_6(g R_z^{-1/\nu r_z}). \quad (7)$$

Differentiating M with respect to h_z in (4), one obtains the susceptibility at zero field,

$$\chi(g, R_z) = R_z^{\beta(1-\delta)/\nu r_z} f_7(g R_z^{-1/\nu r_z}). \quad (8)$$

To fix the critical point, we can define a cumulant,

$$C(g, R_z) \equiv \frac{M_0}{h_{z0}\chi} = f_8(g R_z^{-1/\nu r_z}), \quad (9)$$

similar to the Binder’s cumulant in the finite-size scaling [27]. As C is a function of only one independent variable, its curves of different R_z intersect at the critical point h_{xc} or $g = 0$ at which C becomes a constant $f_8(0)$ independent on R_z .

After fixing the critical point, all the critical exponents can be estimated. For example, $\beta\delta/\nu r_z$ and $\beta/\nu r_z$ can be estimated respectively from (6) and (7) by fitting h_{z0} and M_0 for a series of R_z at $g = 0$. Similarly, from (2) at $T = 0$ and $L = \infty$ $\beta/\nu r$ can be estimated by fitting M for a series of R at $g = 0$. From these three exponent ratios and the scaling law [1] $\beta(\delta + 1) = (d + z)\nu$, one can determine all the critical exponents.

Application to the transverse-field Ising model.

As an example of the FTS method to determine critical properties, we consider the one-dimensional (1D) transverse-field Ising model whose Hamiltonian is

$$\mathcal{H} = -h_x \sum_{n=1}^N \sigma_n^x - \sum_{n=1}^{N-1} \sigma_n^z \sigma_{n+1}^z, \quad (10)$$

and has been realized in CoNb_2O_6 experimentally [28], where σ_n^x and σ_n^z are the Pauli matrices, h_x is the transverse field, and the Ising coupling has been set to unity as

our energy unit. The model exhibits a continuous QPT from a ferromagnetic phase to a quantum paramagnetic phase at the critical point h_{xc} (and so $g = h_x - h_{xc}$) at $T = 0$ [1]. The order parameter is the magnetization $M = \sum_{n=1}^N \langle \sigma_n^z \rangle / N$ for the N spins with the angle brackets denoting the quantum and/or thermal average. As a method to probe the transition, we add to \mathcal{H} a symmetry-breaking term $-h_z \sum_{n=1}^N \sigma_n^z$.

We demonstrate our approach at $T = 0$ at which some exact results are available for comparison. We solve the model using the time-evolving block-decimation algorithm [29], which is capable of treating large system sizes. We determinate the critical point in Fig. 2 and apply it purposely to determine the critical exponents in Fig. 3. The good agreement of the results collected in Table I shows the power of FTS. In Fig. 4, these results are verified by scaling collapses, which provide also a method to estimate the range of the FTS regimes. Similar scaling collapses have been reported [13, 14, 23–25], however, here the critical exponents and the crossover regions are estimated by the FTS method self-consistently.

Nonequilibrium FTS at finite temperatures.

Having successfully demonstrated FTS at $T = 0$, we now turn to nonzero temperatures at which most realistic experiments operate. In order to study thermal fluctuations and examine the general nonequilibrium FTS scaling (4), we have to consider an open many-body quantum system interacting with a heat bath [30]. The state of system can be described by a density operator ρ according to quantum statistical physics. For weak system-environment couplings, after assuming Markovian and tracing over the bath variables, one obtains the master equation for the density matrix ρ in the Lindblad form [21, 22],

$$\partial\rho/\partial t = -i[\mathcal{H}, \rho] + \mathcal{L}\rho, \quad (11)$$

where $\mathcal{L}\rho = -c \sum_{i=1, j \neq i}^{N_E} \beta_j (V_{i \rightarrow j}^\dagger V_{i \rightarrow j} \rho + \rho V_{i \rightarrow j}^\dagger V_{i \rightarrow j} - 2V_{i \rightarrow j} \rho V_{i \rightarrow j}^\dagger)/2$, c measures the coupling strength between the system and the bath, N_E is the total number of the energy levels, $\beta_i = \exp(-E_i/T)/\text{Tr} \exp(-\mathcal{H}/T)$ with E_i being the i th eigenvalue of the \mathcal{H} , and $V_{i \rightarrow j}$ is the thermal jump matrix whose element at the j th row and i th column is one and zero otherwise in the energy representation. $V_{i \rightarrow j}$ fulfills $\beta_i \rho_E V_{i \rightarrow j} = \beta_j V_{i \rightarrow j} \rho_E$, where $\rho_E \equiv \exp(-\mathcal{H}/T)/\text{Tr} \exp(-\mathcal{H}/T)$ is the equilibrium density matrix operator whose eigenvalues are β_i . This can be regarded as the detailed balance condition for $V_{i \rightarrow j}$ in equilibrium.

The meaning of (11) is quite clear: If the second term on the right hand side is neglected, it is the quantum Liouville equation contributing quantum fluctuations to the evolution of density operator ρ ; while if the first term on the right hand side is omitted, the diagonal part of the remaining equation is just a thermal master equation contributing thermal fluctuations to ρ . The Lindblad equation (11) thus is a real-time dynamic equations

which integrates both the quantum and the thermal contributions. It has been widely used in quantum optics [31] and relaxation processes in open quantum systems [22, 32].

In order to show that nonequilibrium FTS does valid for $T \neq 0$, we solve numerically equation (11) for the Hamiltonian (10) along with the sweeping probe field by brute force. We find that the order parameter M_h can now saturate correctly with thermal fluctuations. Moreover, the results displayed in Fig. 5 show clearly the validity of the FTS scaling form (4).

DISCUSSION

We have verified that the theory of FTS indeed provides a unified description of the driving dynamics near the critical point from the realm of time. We have also determined the critical point and critical exponents accurately according to the scaling forms of FTS at $T = 0$. As mentioned, in the FTS regime, the effects of the temperature are suppressed by the nonequilibrium driving, i.e., thermal fluctuations or heating of the low-energy critical modes are governed by the driving time scale τ_d in the nonadiabatic nonequilibrium FTS regime. Only at higher temperatures do the thermal fluctuations take over and the QCR emerges. This ‘dynamical cooling’ effect allows one to determine the critical properties at finite temperatures too. As experiments are always operated at finite temperatures, the FTS method can thus be used in the experiments directly.

In QPTs, the Hamiltonian determines not only the partition function, but also the time evolution of any observables. Further, the imaginary time acts as z extra dimensions in the vicinity of a quantum critical point through the quantum-classical mapping. These place special importance on the dynamic critical exponent z . Note that z can be a noninteger for some frustrated systems [33]. Traditional methods employ usually a finite size in the imaginary-time direction and utilize finite-size scaling to estimate z [33]. In contrast, as the scaling forms shown, z enters the nonequilibrium scaling naturally. Thus FTS provides at least an alternative method to estimate z directly from real-time scalings.

We have used the Lindblad equation to describe the thermal behavior near the critical point. Note that it is a challenge to study the time evolution of nonequilibrium systems with many degrees of freedom even in isolated situations [4, 5, 34–36]. Further, none of the analytic, semiclassical, or numerical methods of condensed-matter physics yields accurate results for dynamics in the QCR except for some special systems in 1D [3]. Although here we only solve directly the Lindblad equation (11) for small lattices, the results show that it is suitable for describing the nonequilibrium behaviors at the finite temperature near the quantum critical point. Moreover,

the rapidly developing numerical renormalization-group methods [37], for example, seem quite promising to solve the equation for larger lattice sizes [38, 39].

In conclusion, FTS not only lights up the dark impulse regime of KZM at zero temperature but also sheds light on the QCR at nonzero temperatures by establishing its own regime. It also offers a powerful unified approach amenable to both numerics and experiments to study equilibrium and nonequilibrium dynamics of quantum criticality. We have also shown that the Lindblad master equation can be a valuable framework for studying real time quantum criticality at nonzero temperatures. Although we have studied a simple model for illustration, our approach should be applicable to more complex systems as well. In addition, as our results indicate that the classical theory of FTS with appropriate modifications can well describe quantum criticality, new physics may be in action [2] if it is violated.

METHODS

Solving the Schrödinger equation at zero temperature. The driving dynamics evolution at zero temperature is solved based on the time-evolving block-decimation algorithm [29]. The efficiency of this method depends on the fact that singular values have an approximately exponential decay. After expressing the wave function in a matrix product state via Vidal’s state decomposition, one can cut off the singular values with a prescribed error induced by the truncation. In the present work, we choose the lattice size of $L = 2000$ to eliminate the size effects. The number of singular values are kept at 16 which guarantees that the errors near the critical points are less than 10^{-5} .

Solving the Lindblad equation at the finite temperatures. The Lindblad equation is solved by a finite difference method to second order with the time-interval $\delta t = 10^{-3}$ and the resultant errors are about 10^{-4} .

* Electronic address: zsuyinshuai@163.com

† Electronic address: lichaoh2@mail.sysu.edu.cn

‡ Electronic address: stszt@mail.sysu.edu.cn

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Author contributions F.Z. and S.Y. conceived the project. S.Y. carried out the data analysis and prepared the early drafts. C.L. provided some ideas on understanding KZM and its related behavior and suggested the TEBD method. S.Y. and X.Q. designed the computer program. F.Z. proposed the FTS understanding of KZM, the separation of different regimes, the finite-temperature study and shaped the project. F.Z., S.Y. and C.L. prepared the final manuscript.

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TABLE I: Critical point and exponents for the 1D transverse-field Ising model

| | h_{xc} | β | δ | ν | z |
|-----------|----------|-----------|----------|---------|---------|
| Numerical | 0.999(2) | 0.125(11) | 14.9(6) | 0.98(4) | 1.01(3) |
| Exact [1] | 1 | 0.125 | 15 | 1 | 1 |

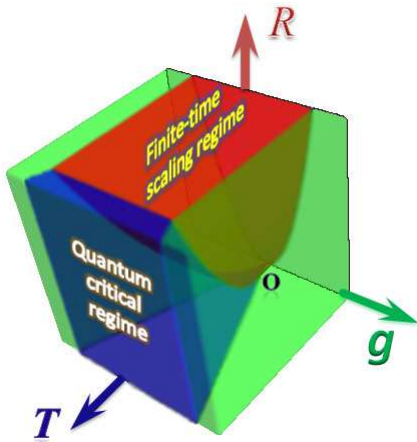


FIG. 1: **Schematic phase diagram near a quantum critical point under an external driving.** Two equilibrium phases (green) in which the reaction time dominates are separated by two crossover domains fanning out from the quantum critical point O . One domain is the quantum critical regime QCR (blue), in which the imaginary time scale or temperature T dominates. It extends from the usual well-known QCR bounded roughly by $T \sim \pm |g|^{\nu z}$ at $R = 0$ to finite driving rates R . The other is the new finite-time scaling (FTS, red) regime in which the driving time dominates. At $T = 0$, it overlaps with the impulse regime enclosed roughly in the curves $R \sim \pm |g|^{\nu r}$, thus improving the understanding of this dark regime of the Kibble-Zurek mechanism. The FTS regime extends to low T and pushes the QCR to higher T . Crossover occurs near $T \sim R^{z/r}$. Because T is only subordinate in the FTS regime, the nonequilibrium FTS provides a valuable access at finite temperatures to the zero-temperature quantum critical point.

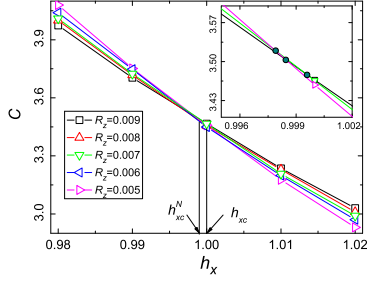


FIG. 2: **Estimation of a quantum critical point.** Curves of the cumulant C (Eq. (9)) of different R_z intersect at the critical point h_{xc} or $g = 0$. Owing to possible errors from the truncation of the singular values in the Schmidt decomposition [29], however, the intersections are slightly scattered as shown in the inset. Nevertheless, the average of all the intersections is $h_{xc}^N = 0.999(2)$, a good estimate of the exact value $h_{xc} = 1$. We choose a lattice size of $L = 2000$, which is large enough for the size effect to be ignored.

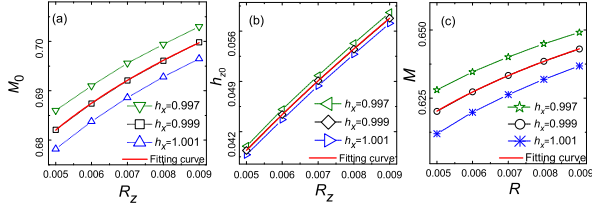


FIG. 3: **Estimation of critical exponents.** In the thermodynamic limit and at $T = 0$, from the finite-time scaling forms (7), (6) and (2), $M_0 \propto R^{\beta/\nu r_z}$, (a), $h_{z0} \propto R^{\beta\delta/\nu r_z}$, (b), and $M \propto R^{\beta/\nu r}$, (c), respectively, at the critical point $g = 0$. Our solutions with $h_{xc}^N = 0.999$, $T = 0$ and $L = 2000$ yields $\beta/\nu r_z = 0.0436$, $\beta\delta/\nu r_z = 0.651$ and $\beta/\nu r = 0.0622$ from power-law fits. Using the scaling law [1] $\beta(\delta + 1) = (d + z)\nu$, we then obtain all the critical exponents listed in Table I with their corresponding exact results for comparison. As the statistical errors of the fits are tiny, we fit data at $h_{xc} = 0.997$ and 1.001 and the largest difference in each exponent is used as an estimate of the error given also in Table I.

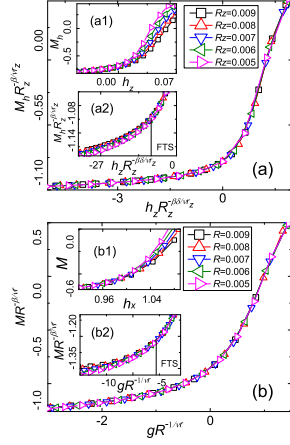


FIG. 4: **Scaling collapses and crossovers.** The original curves in insets (a1) and (b1) collapse as expected when they are rescaled, because they are now functions of a single rescaled variable: $M_h R_z^{-\beta/\nu r_z} = f_3(h_z R_z^{-\beta\delta/\nu r_z})$ at $g = 0$, (a), and $M R^{-\beta/\nu r} = f_1(g R^{-1/\nu r})$, (b), according to the finite-time scaling (FTS) (4) and (2), respectively, at $T = 0$ and large L . This therefore generalize the Kibble-Zurek scaling into the whole FTS regime. This regime, denoted by FTS and delineated roughly by the vertical lines in insets (a2) and (b2), crossovers to adiabatic evolutions. The exact positions are non-universal constants depending on the model. One sees that the FTS regimes are relatively large for the model.

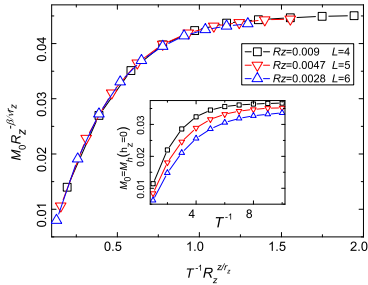


FIG. 5: **Nonequilibrium scaling at nonzero temperatures.** Data of M_0 versus T plotted in the inset for the three different sets of R_z and L so choosing as to fix the value of $L^{-1} R_z^{-1/r_z}$ collapse as expected onto a single curve described by $M_0 R^{-\beta/\nu r_z} = f_3(T R_z^{-z/r_z}, L^{-1} R_z^{-1/r_z})$ for a fixed $L R_z^{-1/r_z} = 0.777$ according to the finite-time scaling (4) at the critical point. $c = 0.8$ and $h_x = 0.999$.